

10<sup>th</sup> Math Sum Angles  
Main

Teacher: Allright. Welcome back guys. Grab a calculator if you need ... So today we are going to be starting the measurement lesson, measurement one. We have four days basically of this, and we're done with the entire semester. So take a look, see what you got, we'll talk about it.

Pledge allegiance, to the flag, of the United States of America. And to the republic, for which it stands, one nation, under god, indivisible, with liberty and justice for all.

You want me to put that on the side? Your guitar.

Student: Uh, yeah.

Teacher: Just stick it over, like on that shelf over there.

Oh Student is going to be there.

Hmm? (to student) Yeah go sit back up, by Student.

Okay, so you can use that. (to student) Use that one.

Allright, I'll give you a couple minutes, find those missing angles, grab a calculator if you need.

[00:02:00] Here. I'll put this down for you.

Here, just use this one. When we do our activity.

Student. Student.

Oh, didn't even tie that one. Very good.

Remember this one, Student? Inside an angles on triangle.

Student: No.

Teacher: Let's take another minute.

What kind of triangle is this? (to student) Remember what that's called? Isosceles. What do we know about the bottom to angles in an isosceles triangle?

Student: They're the same.

Teacher: There you go. They're the same. There's your hint.

[00:04:00]

Student: Is this an isosceles?

Teacher: Why would you say it's isosceles?

Student: Because you just told her it isosceles.

Teacher: (laughs) Why would the diagram show us that it's isosceles?

Student: Because the lines are ... I don't know the word. Congruent.

Teacher: Yes, but how do we know they're congruent?

Student: Because of the little tick marks.

Teacher: Yeah, the little tick marks.

Student: But you just said it was the tick marks.

Teacher: Yes, but the tick marks, right.

Student: Can we just subtract 34 from 180 and then divide by 2?

Teacher: That's a [inaudible 00:04:23] strategy.

Student: You should give us like thirty more minutes.

Teacher: (laughs) We'll do a little more, we'll do a little more. You don't have to draw the triangles, but I'd definitely appreciate it. (laughs) Sure.

Allright, so what kind of triangle is this? (to student)

Good.

Allright, you got a lesson? (to student)

So what does it have to be 34? Do these two angles have to be the same? (to student) Don't have to be. These two have to be the same. So let's think about it, what do all the angles add up to? Inside a triangle, what does every angle add up to? What degrees do they add up to? There you go. So, let's try to figure out what both of these are together, yeah? Allright, let's see what you got.

[00:06:00] Allright, I'll give you guys a little bit more time. I see some good answers here, some good diagrams. Excellent. Good. You got it? Easy. Allright.

Yeah, what do you need Student?

Student: Are we getting the tests back?

Teacher: I have not graded them yet. (laughs) I was very busy yesterday, with these guys apparently. I don't ... If you want to see after class, I can talk about it real quick. Where am I signing?

Student: Where it says math.

Teacher: This one?

Student: Thank you.

Teacher: Allright. Allright, so let's talk about it. This is something we've done quite a while ago, we proved this, what? A quarter or two ago? Interior angles of a triangle. Today, and I know it doesn't say it on the title, we're going to be extending this into polygons, what happens if we have a four sided shape, or a five sided shape, or a ten sided shape, or a thousand sided shape, right? We're going to sort of extrapolate from there and figure out, what are the interior angles gong to add up to. And then of course, just like you did for these, find those missing pieces. Right?

Allright, so for the first guy, what kind of triangle is this? Isosceles, of course. Which means these two angles are? Congruent, sure, the same. Allright so we can label this  $X$  degrees if we really wanted to. Now I know many of you don't need to write the full equation, but to be precise, let's write it out. We know the sum of the interior angles, is always the same, which is? 180.

Allright, so we can technically write this whole equation, and whether you actually did this or not, this is the thought that you guys had when you were doing this. So basically we have  $34 + X + X =$  that 180. And of course, what you guys actually did is, you minus 34 and then you divided by 2. So in the end, when we did that, what did we come out with? Seventy-three degrees.

[00:08:00]

Now because this is an equation, one of the great features that we can exploit is, we can check our answer. So if you have 73, or something else, what if you have 72, or you got a different answer? You can add them up. So you can add up 73, and 73, and 34 and check to see if that is 180. Now again, this one's fairly straight forward, so probably not necessary but that is a way of checking such as on a test just to make sure that you got it right.

Allright, second one, right triangle, same idea. We have our ... we'll fit this on here. 64, plus 90, plus something equals 180. And Student, what did that actually come out to? Twenty-six degrees. Sounds about right. Okay so, fairly straight forward. It's basic arithmetic, sometimes you have to be clever because certain angles are congruent and they don't give you one of those angles, because that would just

give you the answer, so your equation needs to be you know, a little bit more, more complicated. But, in general, it's sort of addition and subtraction. We will get onto some of those equations later on, where it's like  $64+X$  or you know,  $2X-3$ , and you have to make those bigger equations. And we kind of talked about that in our review last class. But for the most part, a lot of the math you're going to do today, is simply addition and subtraction. So not a big deal. It should be pretty good.

No homework last time, yeah? Alright, I have not graded your tests yet. I will do those as soon as possible, if you really want to kind of get an idea of what you have, come see me after class today, we can go through it and I can ... I don't know, I don't think I can grade it that quickly but I can ... We can talk about it and see what you guys actually got. I'll try and do that this weekend, as soon as possible.

[00:10:00]

Okay so today we're going to be investigating and then writing down our last theorem, which has to do with the interior angles of any shape. Not just a triangle, but what about a four sided, five sided, six sided shape, okay? We're gonna start with a little activity, so grab your books, open to a blank page, which I guess is the next page. Grab one of the protractors that I put on your desk. Question. [inaudible 00:10:17] Yeah just do it on there, that's fine.

Allright so, we're going to investigate and if you actually look inside your work book on page 149 there's a little table, we're gonna fill that out later so maybe you want to keep that off to the side, it looks like this by the way. Can we double check? Is it 159? Wish I had a book with me. There is one on 150 but I think there's another one on 149. Check 149. Is that the one? No, there you go. Okay, yes. There's a lot of tables here, we're not going to fill them all out, there's basically two ideas we're going to do. First of all, we're going to investigate. And this is sort of the inductive reasoning. We're going to physically draw these polygons, we're going to measure their angles, which again is not the most accurate way of doing it because we're just going to make them up, and we're going to add up those angles. We're gonna try to sort of come up with an idea, come up with a theory about how these angles are related, inside a bunch of different polygons.

[00:12:00]

Now the book wanted you to do all of them and obviously that's going to take forever, so I just broke you guys up by two polygons. And I've sort of alternated them so you're not doing a million different sides. So on your table there should be a sticky note, it'll say like polygon with nine and five sides, or seven and four sides, or whatever. Just two different polygons, and so what you need to do, is just like I did over here, make a polygon with that number of sides. And then make the second polygon, with that number of sides. Just make them straight, use the straight edge, they don't have to be perfect, they don't have to be regular, they don't have to have any particular length. But just make them and then as accurately as you can, but rounded to the nearest whole number, please measure those angles and add them up. Okay. So we'll take a couple minutes here, see what you guys can do. You can work with the person at your table, but draw them, measure them, and then compare. See if you guys get the same measurements for

each of these items. Alright, take a couple minutes.

Do you need anything? (to student) Sure, grab your things.

They're not going to come out perfect. But get them close enough.

Yeah?

Student: Can I do it on this page?

Teacher: Yeah that's fine.

Make sure you guys have eight and six, okay.

What is that a fourth? So that's when we're going to be constructing, so if you want to do it in your book, you're going to be drawing that shape and then measuring those angles. So for you, you can do the seven and five. Draw a seven sided figure, a nice one. And then a five sided figure, a nice one. You can do the ten and the four, so just draw a ... not, again, not a perfect one. But as nice as you can, so we can measure sharp angles.

So you have your, which one is this? Six sides, okay. So now you need to measure each of these interior angles as close as you can. What you want to do is put that on the corner, and line it up as much as possible. And then you want to see where does this thingy hit. Again, you're measuring this angle. So you're going to start from this side and go up, so that'll be like 130.

[00:14:00]

Yeah please be careful guys, the protractor has two sides to it. One of them counting up, one of them counting down. Look at the angle, see if it's acute or obtuse, and that'll be an easy way to switch to tell which side you should be using. Just be careful with that, because I switch that up all the time too.

You guys got your shapes? Looking cool, alright. Now we're going to measure each of these angles. Get the five one of these, that's totally fine. I'll give you one of these, it's kind of hard to measure when we make them so small. But basically ... Here, put that on here again. If you have this, you line it up with one of the sides and then you sort of see, where does the other side end up. So you can see here, what degree is that? 90 is right here, we got to cunt up a little bit. 93, perfect.

As close as you can, I'm not sure how accurate this is going to be but, we will try it and then we'll do the real way. But this'll be interesting to see how close you guys get.

Yeah that's fine. Now you're going to measure each of the angles. Each of the interior angles. As close as you can, doesn't have to be perfect.

Allright, let's start measuring some angles.

Guys, get at least one of those down and measure it before you start the second one. Just so we can get a little bit of data here.

[00:16:00] You start at the corner, you line it up. So this is your base of your angle, right? So then your other angle is coming off of this side. Now the easiest way is to use another straight thing and then line it up. Now because this looks acute, so what? You're going to use the top part? So 78, sure. No no no, that's counting from this way, that would be 110. So careful.

You guys got one? Oh, yeah the four is easy. Then ten sided ... They actually wanted us to go up to twelve, which ... I'm not sure how crazy that would be. Here's the thing to do, it doesn't have to be all nice, like you can make them go in and out, you can make them kind of do whatever. Although, those are kind of hard to make.

Allright, what'd you find? Okay. 541, oh cool. That's, it is 540 yeah. (laughs) That's really good though. We got close, allright.

These are getting close guys. Let's take one more minute, get at least one down and start adding up those angles. We'll collect as many as we can on our table. And then we'll find the exact ones.

You guys get the ten sided? Is that too many? I guess that works. That's a tricky one.

Five sided one's good.

Yeah, just to a five sided one for me. Let's try that instead. Yeah that's better.

[00:18:00] Allright. Once you get one of your answers, throw it in the table. I'll give you one more minute, we'll collect what we've got. Get at least one down.

Okay so let's start measuring some stuff, yeah? Actually, let's use the edge of this. Pick an angle. On your shape here. Sure. That's fine. Okay put your protractor on it. It's the way you're going to do it, is you see the little anchor right here in the center? That's like the target, you put that on the end, and then you line up this little line with one of the sides. Something like that. And then, you want to see this line. Now since we can't see it going all the way, we want to sort of match it. So something like that, so what angle is that going to be? That would be if you're coming from this side, but see how it's obtuse? So it's got to go the other side. Let's go all the way, so 90, 100. 102 sounds great. Allright put it down.

That's what we're trying to find out. What did yours add up to? I think you're missing one angle. This one, this one. You know what might be easier? Yeah, go from this side, and then you just minus it from 360. Because of the full circle. That

might actually work. Sure. So minus that from 360. Yeah, that'll work. Yeah it's kind of hard to measure the other side.

Allright, do we have a couple angles we can put up here? I know the giant ones are kind of tricky. How about the triangle? We already know that one, yeah? Student, how about the triangle? What is that going to add up to? Remember that one? 180? Allright good.

[00:20:00]

Allright, I saw a couple people do the quadrilateral, the four sided one, although we've already talked about that, so what does that one come out to? 360. Allright, now I saw a couple people doing the pentagon. Isiah you had that one, what did that add up to? 541, but you're thinking it may be 540. Allright, now let's just survey anyone, does anyone have another sum that we can do right now? Anything bigger. I know these were kind of tricky. You guys are going to finish up. What is that? Seven sided? Eight sided. Okay.

There is a pattern here. But it's kind of hard to find. When you're measuring things, you're not getting exact angles because you're just making this up. This is a problem. However, when people had looked at this throughout history, they noticed that there was something happening. There was some sort of pattern. When they took two different pentagons, and they measure the angles, they were all around 541. Around 540. 539. Right there. They were all around the same sort of area.

You get the eight sided one? 880? Okay. They were related in some way. You guys can see as we get higher up here, these numbers seem to get bigger. Which kind of makes sense, there's more angles. There is some sort of pattern. You get another one? For which one? 1020? Okay. Or 1020. Okay. You guys want to see the real answers?

[00:22:00]

It is adding by something. You know what, before I even show you guys this, let's try something else here. Let's try something else here. This is inefficient, because we literally have to measure every single angle. But we already know a fact, we already know something about polygons, at least simple polygons. And that is, triangles have a measurement. So if I draw a triangle and I add up the angles of any triangle, what does it come out to? 180. Doesn't matter how I draw it. Doesn't matter what it's shape, it's sides. No, right? This is a fact that we have proven, and we prove this several different ways. The way I remember was the parallel lines, draw the little triangle in there. But there are lots of ways of proving it.

So here is what we want to do. Let's see if we can take those polygons, the ones you just drew, and divide them up into triangles. Now here's the thing, you can't just divide them up into any triangles, because that will not work. You need to divide them up nicely.

Let me show you an example. We'll take this guy. I'm going to divide this up into

triangles, you guys tell me if I did this right. Not really. Here is the problem. I want the triangles to help me find the interior angles. These triangles, and this isn't even a triangle ... These triangles do not have their vertices at the corners of this shape. Which means the vertices are not going to be able to equal the corners of this shape. I want to draw it nicely. And I'm going to show you guys a trick so you can use this on your diagrams. The trick is, pick one vertices. Highlight one vertices on the shapes that you just drew. Doesn't matter, any one. And then connect that vertices to every other vertices. Now clearly the adjacent ones are already connected so not necessary, but for these, connect it, and then connect it. For this one, I guess there's only one connected. So I'm going to give you guys a minute, this will be much easier than trying to add up all this stuff, take your two shapes, and connect one corner to all the other corners that you can. And then count up the number of triangles. Give it a try.

[00:24:00]

Depending how you drew it, it can come out kind of weird. Just make sure it connects to the corners and you should be okay. Yeah, that works. Sometimes it's hard when you do the concave one. But that's okay. This one is weird. There you go, that's a good one to choose. Good.

Allright so what we're doing, we're connecting one vertices to all the other corners. So the triangles are not over lapping each other. Exactly. Now the thing is, this one is already connected with the side of the triangle, or the side of the shape. But yes, the other ones looking good.

That sounds much more reasonable. Very good. I don't remember the answers, but yeah you were actually closer.

Allright, let's fill out our table. Depending on which one you had, we can use them all and fill this out. Allright, so turn to page 150. So this is the much better way of investigating this, it does not require that you measure any angles just like inside a triangle it does not require that you measure nay angles. Let's fill this guy out. Get our theorem. Allright, so, number of sides, very simple. Does it stop at octagon or did I cut that off? Okay, so we're just going to see the pattern, you can extend this as far as you guys want. Allright, number of triangles formed by the diagonals. For a triangle, I assume that's one. How about for quadrilateral? Two. How about for a pentagon? You guys see the pattern? Okay. Now what is the sum of the interior angles of one triangle? Okay so I'm just going to do that. Now think about this , the triangles make the polygon. The triangles corners make the corner of the polygon. So if I had up all of those corners, in all the different sides, which in effect is adding up all of the angles inside the triangles, I should get the full sum of the interior angles.

[00:26:00]

So for a triangle it's obviously 180. How about for a quadrilateral? How about for a pentagon? Okay, keep going. Fill out the table. Follow the pattern. Much easier than measuring all this stuff.



By the way, you guys will be using this table quite a bit during today so you don't have to calculate it every single time. Make sure it's all filled out.

Do you not have your book? Okay, don't write in this one, but you can use ... What are we doing? Perfect, perfect, yeah. But just so you can see it easier. So basically this and this are the important parts.

[00:28:00]

Allright hexagon? 720. Heptagon? 900. Octagon? Nope. I don't know why. That's not even a time, what am I doing? Allright, by the way, let's just take a side ... Do you guys recognize these numbers? You ever seen these before? Anyone skateboard? You guys know these numbers? Tony Hawk, super famous for doing the 900, yeah? I think on snowboards and stuff they do 1080's and more. Stuff like that. This is something that is sort of familiar to many of you, right? In X games or in the Olympics or whatever. When someone spins on a skateboard or a snowboard or skis or whatever. They don't usually land sideways. That's not usually a good way to land. So they usually are either landing, forward or backwards, which is a spin of 180.

So if you keep adding 180, as we've done here, you get those multiples. What comes after 1080? What comes after 1080? 1260? So they keep going, and obviously you can make this go as long as you want. You can find any of them. But that's kind of tedious to just kind of adding 180 forever. If I said all right I have this, 100 sided polygon how are we going to find the angles? Eh, kind of a lot, so let's see if we can figure out a better method for doing this. Sorry, by the way, here's what we just did. Up to this. You can see the triangles, you can see the number of sides of the polygon. And there is some method here. So if you guys could answer those questions on the bottom of that page, how many triangles would you need for a 22 sided figure? What would the angles be? And then see if you could come up with a simple formula for this. And we'll talk about that in a second.

Which ones are these? Four through seven I guess. Give it a try.

[00:30:00]

If that's the angle ... Or if that's the trick. See if it works for one of these guys, but yeah. Look for the formula. What do you think? 22 sided one. How are we going to get that? Without like making our table go forever. Think about this, think about this. Look at this guy. Look at this guy. So 22 sided, what is the relationship between these two things? What are they doing? Six and four. Seven, five. Eight and six. Is there like an easy way of getting from one number to the other? What can I multiply this by? Ignore the 22, let's find the pattern first. So we have from five to three. What do I got to do to five to turn it into the three? Okay, did that work for the next guy? Okay, so what about the 22? Easy.

What's the pattern here? What do I do to the five to get to three? What am I adding? What do I do to the five to turn it into the three? Oh minus two. How about the six to get to the four? How it works. How about the twenty two? Get them.

You found it? So what's your formula? Yep, yep. So how are you going to write that? Okay, and then what are you gonna do with that to get the answer? Got it.

Yep. Alright you got it? Oh, giant number, okay. Okay.

Allright, I want to see some good equations here.

[00:32:00]

Allright, how'd you get 21? What's 22-2? Do we have one more? So that's the question. That was a good thought, that's the question. Let's look at our pattern here. So when I had four sides, what did we do to the four to turn it into a two? Like what math did we do? I think you said it originally. Minus two. Did you then add another triangle back on? No. Okay, so no not necessarily, you were right the first time. Cool.

You guys got it? Wait until Student gets back.

So how many triangles did you guys make?

Allright. Let's take a look. How many triangles? Twenty. As you guys notice, the pattern is minus two. There's a reason for this but we don't have to go into it today. So the sum of the interior angles? 3600, right? You take that 20 and you multiply it by the 180. Alright, so let's write down our theorem here. Of course, you want to put this in your book afterwards so you can use it but let's just write it down here. So basically, the theorem for the sum of the interior angles of any polygon, the call it an N sided polygon, I saw many of you using X, totally acceptable, but the official one is N. So it says, the sum of the measures of the interior angles of a polygon with N sides, is  $(N-2)*180$ . That gives you the number of triangles and then you times it by the 180, which is the angles inside that triangle. Alright please put it down. And then let's practice. So there's your formula for the day, now you can see how we got it. That is what we're going to use for the rest of our lesson.

40 ... Okay we got time.

[00:34:00]

I should probably put this on the board.

Allright, let's give it a try. I've got a polygon here, some polygon, let's find the missing angle. Work with your table partners, you've got thirty seconds. Let's go. You can use a calculator of course. Let's see what you got. So what's it going to add up to? How many sides? Check your table. Let's see what you got.

Easy. You minus from 360? (laughs) Yeah, easy.

Allright. Twenty seconds. You got them? Let me see. What? It's actually like a pen. It looks dried out, yeah. It looks like it's leaking and stuff, I don't know. Okay if you've had that since seventh grade then it's definitely not going to be working.

(laughs)

[00:36:00]

Allright, what'd we get? 64. Allright, so what kind of shape is this? Quadrilateral, you count the sides, therefore we check our table, 360 degrees, add those all up, minus them from 360. Here's another trick that I like to do guys. Sometimes I forget the table, sometimes I forget the stuff so here's what I like to do. I will just literally draw the triangle on the paper. So now I can see, oh yeah, two triangles,  $180+180$  right? And I'm done. That is a much nicer way than having to reference a table or memorize something, just generate it for yourself as we go forward.

Allright, we've got one last thing to talk about before I let you guys practice. We have some special polygons, we have some polygons that are nice. These are called regular polygons. So a regular polygon is a polygon that has equal sides, equal angles, we know the equilateral triangle, we know the square. But of course all the other polygons have the same property. They don't really have fancy names, we just call them regular polygons but you can see we have a bunch of different types. They're all equal, obviously their angles are getting bigger as we go. And they have some special properties here. The main property is their angles are equal, like we said. Which means, we can find the measure of a specific angle without having to actually measure all the other ones. That works out pretty good. Allright. Let's see if we can figure this out. I got two polygons here, work with your table partners, find me the angle or the measure of a single angle in that polygon. Remember they are all congruent. So very similar to how you did the equilateral triangle, let's see what you guys got.

[00:38:00]

Student, easy huh? Perfect. Got them. So what did it come out to? Good. For the first guy? Okay. Let's find B now. allright, I see people have got the answer already. Use your tables, references.

What'd you find Jordan? That looks awesome. I don't remember the answer to this one, but that looks about right. How'd you find this one? What'd you divide by 8? Perfect. Got it.

Allright, I see some answers already. Allright, Dustin, A? 72? How'd you get 72? Oh okay. So you're on the right track though. You're on the right track. So he had a sum, and he divided it by a certain number of sides. I think you just had the wrong angle. So let's fix it. What is the sum of this type of shape? Is it 540? So five side shape, 540. And we divide it by? 5. Because it's five sides. They're all equal, and that gives us? 108. Very good. So 108 should be for A. And then for B? 135. So whatever this is, was it 1080? The full one. Something like that. And that adds up of course to 1080 and you divide it by 8. This is the other thing that you guys will see, they will not give you any angles. They will give you a shape, and you need to do the division yourself to find those angles. These are kind of nice though, right?

Now in the book they have this table ... I'm trying to think if we have time to do this. I think we're going to skip this for now, you can generate them as you go. We

[00:40:00]

may have time at the end to fill this out, but I don't want to do it right now. Basically they want you to kind of generate those interior angles. Now let's just put a few of them in here since you guys know some of them. We said the 8 one was ... What was it? 135? For the eight sided one, we just said 135. And then the pentagon was? 108? How about a four sided shape? You guys know this one. What do we call a regular quadrilateral? What's the fancy name? They're 90, so what do we call the shape? They are 90, but what do we call that shape? Of course, a square. And we have the triangle, you guys know that one. There you go. Allright. Again, don't worry about the rest of these. You can generate them as needed, we'll use them later. If you want to fill this out when we're doing our practice, that's fine, I just want to move on for a second though. As we go forward, you can find it each time. Basically you just have to find that sum and then divide it by  $N$ , whatever the number of sides are. Or the number of angles in this case.

I want to talk about one last thing, I saw some of you guys drawing things that look like this. Is this a polygon? Yeah, that's fine. That's a polygon. How is it different than the normal kind that we draw? Something about a shape, yeah? Something about its angles actually. Yeah, it has these reflex angles, right? So the reflex angles are bigger than 180, and they're interior like that so they're kind of hard to measure. But they're still angles, and this is still a polygon. This is not a polygon. When you draw it so it overlaps the corners, that suddenly becomes separate polygons, right? A polygon must have a continuous area, which means that all the area in here must be connected to itself. I'm going to show you guys a little animation here, just to prove to you guys that this worked. What I've always learned, or what they've always said ... I was looking at a text book, they always say, use these polygons. Right? Use a convex polygon, don't use the concave ones. And I'm like, well why not? It's still a polygon. So I found this little program that's going to animate this, and let's see. Let's see if it works. So I want you guys to look down here. These are the angles, the computer is just measuring them, and then it's adding up those three numbers. So let's see what's happening here.

[00:42:00]

Okay, angles are changing. What's happening to the sum? Rock solid. You can make it really tiny, really big, but we knew that. Triangles, obviously. Let's look at another type of shape. How about this guy? You can see the sum, 540, like we know. Let's mess it up. Not changing. Even if we make it crazy like this. Rock solid. This is the cool thing, doesn't even matter what the shape looks like, unless ... Did I do it in this one? I can't remember. (laughs) That works too. Unless you overlap it, like this, and now suddenly it can't calculate because it's two triangles. Right? It doesn't work. You guys can see. And of course, for any shape, this'll work. Here's the 1080, the eight sided shape. Make Pacman. Any crazy shape that you want. All the angles move, but you can see, as one of them gets bigger it sort of forces the other one to get smaller. Right? No matter what crazy shape you make, it will still work. Unless, of course, you cross one of them to not make a polygon. I don't remember if I do that for this one. Allright, you guys can see. Even that one would work. They usually don't give you ones that look like this, but you guys get the idea. It still would work even if you had it.

We'll do two more things here. Guess who needs help. (laughs) Guess who needs help making a little bench. Your favorite. Grandma's got a bench, got to go with the gazebo, you guys remember the shape of the gazebo? Was it a hexagon? I think it was a hexagon. So she wants a bench to go in the middle, to kind of match that gazebo. Here is the bench. You have to cut the pieces of wood to nail together here, so with your table partners, I want you to find me this angle. What angle would I have to cut that, so these two things would kind of join nicely? You guys can get this pretty quick actually, since you know all the other stuff. Alright, give it a try.

[00:44:00]

What is this black sort of ... This angle right here? That I have to cut down the side there. So maybe draw the hexagon from the top if you need to. See what you guys got.

I don't actually remember what the answer is for this one. What was the ... Oh we never did the hexagon.

Hexagon shaped, see what that angle is going to be.

Okay. I'm going to pass out something while you guys are doing this, just keep it on your desk.

What's that? I don't remember, maybe. Why? Were you going to say 60? Okay. Okay. Oh interesting, that would work. Very good.

Give that to Student.

How's everything coming? Are we going to find this guy? We know it's a hexagon ... Maybe let's find this. This may be a strategy. What if you found this angle? The big angle. See if you can get that guy first.

[00:46:00]

Allright, Natalie, you got it? 60. How'd you get 60? Okay. That's the full interior angles. No you were right the first time. You took the 720 which is the interior angles of a hexagon. You divided it by six. That gives you this angle. This outside angle. But that's not the angle we're cutting with our saw. So how'd you get the actual angle? Exactly. And you guys remember we talked about this, when you're doing like trim or whatever in a house ... We watched that video. You want to make it equal, no matter what the angle is, you got to bisect it. So the easiest way of course is just to know the angle and divide it. But remember we had those paper tricks, or the compass tricks and everything? So that one works too. Alright so, this a real life application, I've had to do stuff like this when I've been building little home projects and stuff, and of course grandma appreciates your guys help.

One last piece, what if we have two polygons that are stuck together? So lets do this one together. And then we're going to do some group work and present a few

problems at the end here. Let's figure out what's happening here. We got two regular polygons. We have a square and a hexagon, we want to find this angle. This sort of outside curving angle. What should I look for first? The interior angles, sure. The other side. How about for the square? We got, we got 90 degrees right here. How about for the hexagon? We just did this one. 120. So what's next? What're we going to do? How'd you get that? So you added up  $90+120$ . And you got 210. So that's, that's this angle. And then what? How'd you get 150? Oh, wrong color. So you went  $360-210$ , and you got 150. Allright.

[00:48:00]

So this is the other skill and we talked about this last class, many times you can find part of a circle and then you can use a little subtraction to find the other missing part of the circle. just like when we had a linear pair. You can find that missing angle that adds up to 180. These angles add up to 360. And so you can sort of generate that by subtracting it off the end.

These are the basic skills that we need to be successful at this, we're going to be solving some problems, so grab the paper in front of you. I know it's kind of small, so if you need room please use your book, this is going to be our practice. Let's take ... What did it say? Allright. We'll do about 15 or 20 minutes. We'll see how long it takes. I don't know. There's a couple problems on here, there are some equations so please be careful. You may want to write the full equation. I know you guys are good at just sort of finding some of this stuff in your head, but if you could write the full equation out in your book, that may help you for the ones with a lot of X's in them. Just be careful with that. I'll turn the radio, we have 15 minutes let's say. Please work on your hand outs, work quietly with your table partners, and be ready to present a couple at the end. We'll throw some up on the board.

Is it allright if I turn the radio on a little bit? They like to work with some music so ...

(music plays)

How you doing? Now would actually be perfect, so just take this, I'll write you a pass.

Just finish that up at home, then there's a homework page right here that you can work on.

[00:50:00]

Allright. How you doing? So what are you thinking? Which one do you want to start with? Sure. B. Oh D. Excellent, okay ooh interesting. Basic parallel lines, okay. So we want to find X. You see you've got parallel lines, is X equal to any of the angles over on this side of the parallels? Got to be, they're parallel lines. Ignore this then, don't worry about the polygons. Just parallel lines, which angle is X going to be equal to? Which one? Right here? This side. Oh perfect, because it's corresponding, okay. So we got that guy. Any others? This side too, okay. So maybe this is your strategy. Maybe you go from that guy instead. Allright, show Student.

That would be your strategy to find the sum. Yes. If you want to. Then you don't need to do the triangles. You definitely don't need triangles for that. Here's the problem. If you start putting triangles in here, your chopping up that angle into something that we don't know. Into like two pieces, you know what their sum is, but you don't know what each separate piece is, so you don't need that. What do we know about all of these angles? What you're saying is that, this and this, equals 403. Right that down. Like I said, sometimes ... Like this one, you don't need to. This one you literally need to write an equation. Yeah, wherever you have room.

[00:52:00]

Allright. You doing okay? Is it 180? Lets think about it. How many triangles are inside this shape? Each triangle is 180. So what are they all together? You don't have to do it in your head. You've got three triangles, they're each 180. That's a great way to do it. That's what they all add up to. So now can you find this angle? Yeah let's do it.

How you doing? Excellent. So the whole things adds up to ... Oh. Careful. Careful. Remember the thing we talked about. Within this shape, we have how many triangles? Four. Right. Two less than the number of sides. We have four triangles. That's why you have to do the  $180 \times 4$ . Otherwise it'd be too big. Give it a try. I think I actually have one, let me see. Ah yes, here you go. Just give it back at the end.

So what's the 403? That's what these guys were? Those four, minus from 900, gives you that. Okay. Excellent. You're saying that this guy, and this guy, and this guy equal that. Okay. Let's find the N. Write that equation.

How you doing Student? Oh yeah, go them. Let's look at this guy. This guy's pretty good. Allright, what kind of shape is this? Heptagon. Seven sided okay, so what do they all add up to? Okay. Start minusing.

[00:54:00]

You good? What kind of shape is this? Yeah. Use your table. Just all go from the same vertex though.

You're saying, ignoring this as a triangle and this as a separate shape like the whole thing? Let's see. One, two, three, four. Sure. Here's the only problem, you know this angle ... Or actually, you're not even counting that, you're counting that as a whole side. So what is this angle? This is 90. But what is this part of it? You're going to have to add that part to the 90. Did you find that? So kind of like our bell work. That was a great strategy, doing it in one step, but finding this part first, and then doing your whole subtraction. Yeah, that would totally work. Do they triangles separately, and then put your numbers in and start doing it from there.

How you doing? Which one you looking at? Oh easy. Yeah that's actually a good one, you can do that one for a second one. A lot of regular polygons in there, works out good.

How you doing? You can do this one first, that's fine. Cool.

[00:56:00] How you doing? Which one you on? You can jump down there, that's fine. Don't forget some of these. I know there's some equations in there, never fun. What's your strategy? Oh you already found them. Oh perfect. And you did ... You added them. Yeah, that's good.

Let's see you did ... Wait, how'd you get 360? Can we try that again? What's  $180 \times 4$ ? So we have a ... What's this? A six sided shape? So it's a hexagon? Just try it on here, what's  $180 \times 4$ ? Just try it. See what you got. There you go. You can see that was this part on your table. You can do it ... Like you showed. You can draw a little pictures, draw little triangles, and then just times them. Okay now that's the thing you're going to divide.

Which one's tricky? This guy. Okay so what kind of shape is this? Count it again. How many sides does that have? How many is it? Seven. Okay so that would be? Heptagon, yeah? What does that add up to? 900. That's where you can start. You know everything adds up to 900, so you can write this nice big long equation, that has all of these adding up to 900. You probably want to do that in your book since there's not a ton of room.

[00:58:00] What did you find? I like how you divided it up though, excellent. Okay. What kind of triangle is this guy? What kind of triangle? It is isosceles. Just like the bell work. Here's your first strategy? Find these guys. Well all three of these equal to 180. So let's figure out what one of them is going to be. You see that Student? Let's figure out what both of them are together. What do you do with ... To find that? Close. Not add it by 180. There you go. Try it, tell me what you got.

Allright. One sec. Let's take fourteen more minutes. Help each other out. We'll present a couple of these, at that point.

Let me see. Both of these together are 160. How big is one of them? Because they're both the same remember. What do I do to 160? These both equal 160. Together they equal 160, and they're both equal to each other. What do I do to 160 to find one of those? There you go. What do you divide it by? How many angles are there? The ones we're talking about. Yeah, what are you going to divide it by? Now that you have that, you should be able to use the whole shape.

[01:00:00] Allright, you guys had a question. Present it? Which one's that? Sure, if you can show the full equation. That's fine. Okay. Give it like 10 minutes and I'll have you guys write it. Which one? I don't know do you? If you don't need it, don't use it. Sure, whatever you need. And then you got that 90. But then we have two missing pieces. Shoot. We basically want to find this angle. I don't think it's equal to K, necessarily. We're at this point where we need to find this angle. Maybe we should use the triangle. This is 180, so maybe we need to find this angle then. Interesting. Got them.



Smart, mart. I like it, I like it.

Which numbers do you subtract? These guys? But just these four, yeah? Now you know whatever's left ... Was it four-something? Now you know 403 is equal to, this guy plus this guy plus this guy. So write it down. That's when you have to actually write the equation. You can't just do it in your head.

[01:02:00]

You're done. You're drawing so you're done. Which one's tricky? First one. Basically what you have here, you have 403. What is 403? What is that equal to? The angles with N on them. Let's write that down. Here, do it in your book so you have room. Let's literally write that equation. Remember you can do some of these in your head, like the second one, but some of them you literally ... You need the equation. What are we going to write? 403 equals. So it's the sum of that stuff, yeah? Let's just write that down. Put a bunch of pluses in between. See what it comes out to. Okay, looks good. So what do we do with these types of equations? You pretend in N is X but ... Which things could you put together? Yeah, let's do that first. And then deal with the numbers.

Make sense? Good, good.

Please remember guys, you can always check your answer at the end. Once you get an angle make sure, is it obtuse? Is it acute? Does it make sense with the type of polygon that you were given? See what you've got.

You're going to use this shape. Now that you have 58, how big is this angle? No no no, it's a straight line. This side is 58, what's left over? What does a line add up to? Just a straight line. What is the angle of a straight line. There you go. Minus it. Find it. Then you can use your little triangle thing.

Is that necessary? You can, of course, but does it need it? It's easier because it made a triangle? Yeah, sure. Of course. You just won't know that top angle. Do you know this angle? Do you know this angle? Keep going. Well I don't know, is it 120? I don't think it's 120. What kind of shape is the big guy?

[01:04:00]

What kind of shape is that? Count it up. Seven sides, you know what we actually call that guy? That's the one I always forget. We got a heptagon, what do they all add up to? 900, okay. Can you write an equation? Maybe that has equals 900 in it somewhere. It's going to be really long, but it'll have some pieces in it. Yep, put them all together. Of course, definitely the N's because we need that in our equation to solve it yeah? It's going to be long. If you want, I know some people added the numbers up first, like the separate numbers, and then put the N's just to make it a bit shorter. So that's a little short cut you can do.

Seven minutes guys.

Which one's tricky now? E? I like that you labelled this triangle, you have the equal

angles here. That is definitely the first step I would do. Sort of like the bell work. See if you can find that missing angle. And then you can sort of start dealing with this entire shape. Ignoring the triangle, once you get this little angle in here. Look back to your bell work and see if you can figure out what each of those angles are.

[01:06:00]

You found them? You found, what was this angle right here? You had 58 over here, you see this one Student? You doing the same one? This is 58, now we're trying to find this big angle. I think you can just leave it. I don't think you need to divide it by two. I think we can use the whole thing. Yeah just leave it. So now here's what you have. You have 90 degrees, you have 122, you have 88 and you have 96. Now let's find K. What do all of these add up to. No, there's three triangles. What's it going to add up to? One sec. You can look at your little table here if you want to. Three triangles. There you go. Start minusing.

[01:08:00]

You got some parallel lines. That's true. Why don't we start labeling some of those unknowns as like X and Y and stuff. Sorry, X and Z then I guess. Now, are there any other angles in this diagram that are equal to X or Z? It is, very good, let's label it Z. Sure. We saw the problem, because we have too many angles. So we have to see if we can figure out some of these angles. Maybe we can write something a little bit different here. Yeah, some sort of equation. There is definitely 180 in there, yeah. We don't. Can we write X as a Z? Like using the letter Z instead of X. What is X as it relates to Z? Sure. I don't think you're going to do that. It's the same.

I have a lot of white out. (laughter) I guess that's good, but no. What I was thinking is something ... And again, I don't know the answer but something in terms of ... You know this is Z, and you know this is related to Z. It's not just another variable. This is the same. Let's say Z was 80, what would X be? If Z was 80, what is X? How'd you find that? So you minus it from 180. That's the strategy that I'm thinking. I'm just making up something. It kind of looks like it could be 80. But basically you don't have to write a third variable you can just write  $180 - Z$ . That may help you. That may help you. I'd give it a try. I don't know what the actual answer is. You guys tell me.

Allright we have 15 minutes. You want to, you want to start writing something? What are we doing? Which one? Wait. Which one is this? This one. Which one are you doing? You can write smaller ... Would this help? Here, try writing. I just want to see, we can change it. Does that help? Is that tiny? You can zoom in, if you want to make it very precise. And then we can zoom in for the guess. And you can change the colors too as you go.

[01:10:00]

Allright, let's take about four more minutes. Looking good guys. If you finish early, feel free to start your homework. We'll present a few of these and talk about them at the end.

No, homework is 152. But, start on this one first.

Allright, what do you got? I don't actually know how to do this one either. I'm trying

to figure out what the trick is with this. These aren't parallel are they? No I guess not. So here's the only thing I can think of. What's the relationship between those two angles? Got the parallel lines. Got some angles. I don't know maybe. Try them. I mean don't just change it, but see if it's equivalent in some way. So what do we call these types of angles that we have? Kind of look at it like this. We have parallel lines, we've got the transversal, what do we call those lines? Close. Interior, there you go. So they're equal right. That's the strategy I'm thinking we've got to go for.

Allright which one's tricky? So you know what it adds up to? 900. And you minused the angles they gave us. Basically, what you're saying is this is equal to the three remaining angles. Let's make an equation, they add up to this. It's going to have a bunch of N's in it, but that's fine, you can put them together and see what happens.

Got the same thing? Cool, let me know if you have questions.

[01:12:00]

Let me see what you got. Which one's tricky? Which one? Which one? This guy. No this guy, come on we can do this. Don't be afraid of the N's. not even needed. How many triangles fit inside here? Yeah start with that. Allright, how many? Or look at the table. So what are the angles? Add them up.

There's five triangles, each of them is 180. So what is the total sum going to be? Nope. Five triangles. Okay so 900. Yeah that's fine, thank you. So, 900. We know the all add up to 900. So you can write this nice long equation, all of these adding up to 900.

That is really zoomed in. Oh actually, what you ... Basically if you hit this button, it will lock it to the view. So you'll want to put it on the view that you want, and the hit that button. It'll lock it. We can zoom in later. Don't even worry about it. Probably is wrong. I don't actually know how to solve this one. We need more information. It could be. Let me see what other people got, I don't even know the answer to this one. It seems reasonable, but then that might be too big. Well that's obtuse, so that would work too.  $45 \times 3$ . I'm going to see ... Let me see what other people got.

[01:14:00]

You got C? Let's see. I know, Student did you get C? I don't know if this one's even possible. I feel like there's not enough information. What would make this work? Did they forget ... If these were parallel would it work? Like the other two? Yeah. Okay. I don't know if they meant that, but let's put that.

Can you ... can I do something real quick? I'm going to talk with you, just make these two lines parallel, put the little marks on it. Okay.

Guys we've got ten minutes. One quick thing I want to add. I don't know I this is how it's supposed to be, I don't know the answer, but ... Can you guys zoom in on C? If you guys look at C, I don't know if there's enough information to solve this one. There should be but I feel like I'm missing something. So just to make this

work, the way that we've talked about it ... If you can make these two lines parallel, then you can solve it. Then you have more equivalencies that you can then write. If you have these then of course this angle is congruent to this angle. So this would be  $180 - 3Y$ . And suddenly you have enough variables to solve for this. So that's what I would say just to make this a little more doable. I'm not exactly sure what they wanted us to do for this one but, the other ones come out pretty nice. Feel like we're missing something for that one though. Probably my mistake when I made the copies but I don't know, we'll see.

Let's give it one more minute, finish up your last problem, we're going to present a few of these on the board, we're going to talk about them. And then we will do our exit pass. Doing good guys.

[01:16:00]

If you could put your compasses ... Or your, protractors back on the sticky note for the next period, that would be awesome. I think there were two on each one. Something like that, if you can do that by the end, that'd be awesome. Thank you guys.

You can zoom in on that. Do we want to do one more? You want to do another one? Alright, you got it, that's fine. Write it down. Will you finish that one? Yeah, well, let's do that together though. You guys can write it live when we're talking about it. Yeah we'll do that at the end. When we're presenting just zoom in so we can see exactly what's happening.

You got them? What'd it come out to? Oh we got an equation. So you ... Where's the equal sign in this equation? It's an equation ... What did we say all of these added up to? Exactly. The whole thing equals 900, then you start subtracting stuff off to get those numbers down.

Allright which one are you on Student? Same thing. Look at what equation he's making. Alright, give you a little more time.

Allright. We doing better? Making sense? Which one's tricky? This one? Hurry and finish that one. Okay.

[01:18:00]

Allright guys. Let's talk about it, eyes up front. Can you zoom in to B for us right now? Wait, didn't we do that one? Wasn't that one of my examples on the notes? Talk us through it, how'd we find X? So six sided shape, what's the fancy name for that? Hexagon. They have six sides, so you found the 720 which was the sum and then you divided by six. How did you know that you could divide by six? We can't do that for example ... Here, zoom out of it. What's that? The theorem had to do with a regular hexagon. How do we know this is a regular hexagon? The angles are all X. Now this doesn't actually have to be a regular hexagon. I could if I really wanted to, stretch this middle side ... Move it down. I could stretch this middle side out to be twice as long and the angles would not change. So this is not necessarily a regular hexagon, but the regularness that counts, the angle sums, or the angle

equivalencies those are there. So we don't really need to worry about the sides. It doesn't have the sides labeled so it's not necessarily a regular hexagon.

[01:20:00]

Show us F. Can you zoom in on that? Allright let's take a look at F guys, this is more of the homework questions that you guys will see. Basically there are a bunch of variables inside this polygon. So let's see if we can figure out what is happening here. So first of all, what type of polygon is this? Let's count them up. Is that also a hexagon? Six sides, okay so hexagon. Nice easy one. Is this a regular hexagon? I assume not. Nothing is labeled, so. What did you do? So that is the sum of the interior of a hexagon like we said. Excellent. So X is 14, but that is not any of the angles. So how did you find the actual angles after that? Very good. And you got all those angles. Did you check, did they all add up to 720? That is something you could do, again, I trust your work. But that is something you guys could do to find the total sum to see if your answer actually makes sense.

Now a couple things that I point out here that I really like, I really like that you have the full equation, not just pieces of it. You can kind of keep yourself organized. I really like that you kept everything in parenthesis, so you didn't have to worry about negative signs or stuff spilling out, and then you guys can see ... A couple steps were skipped here but that's fine, combing the like terms. You want to get all of the X's together so you can divide off that coefficient and of course we move the 8 first, divide it, and then it comes out nice. I guess it doesn't have to come out nice, but you know, it's a math problem in the text book so I assume it's going to be okay.

Allright awesome guys, did we have one more or are we good? Actually we are almost out of time.

[01:22:00]

So let's just go through the bottom here. These are the answers, so let's see what we got. Can we zoom ... There we go. You guys can see most of these angles such as B, A, C, E. These are the interior angles we've already calculated. So those are trivial. Some of the other ones though you have to do a little bit more math. For example, I, we've got this sixty degree angle right here in this equilateral triangle. And I, is sort of that outside angle which is sort of in between the octagon angle and the triangle angle. So you kind of just do a little subtraction. This is a trick I want you guys to be thinking about, and we mentioned this last time. Many times when you are finding a missing angle, they will not label the thing you actually need to find. They want you to find I, but they don't tell you, oh yeah but you have to find X over here first. You've got to find the big angle Y second. They don't tell you that, you have to sort of find what you want. And this is a balancing act because you could find fifty different angles. And forty-nine of them wouldn't be useful to you. You could find this angle, and this angle, and this angle, and this angle, but they're not really useful. So you want to concentrate it around the area that you actually have here.

Awesome guys. Grab this hand out, you can hold it for now, finish it up if you have

not. Grab your notes. Let's do our exit pass. Actually if you could close your work book, I want to see if you guys can do this without looking at the table. Just make sure you put your calculators back at the end. If anyone needs to look at their test, come see me after class. Alright ready? Go. Show me before the end.

Okay perfect time you guys. That's what calculators are for, come on.

What is this shape called? Nonagon? I don't know. Count them up. Just count the number of sides.

[01:24:00] So how many sides? How many sides? So how many triangles? Inside? Remember the trick? If there's nine sides there's? How many triangles? Remember the minus two that we did? So if there's nine? Seven triangles so find the angles. Let me see. That seems really big. Good. Good. Good. Good. Good. Good.

Allright, how many sides? Nine. How many triangles? Seven. How many interior angles total? Sorry, the sum of the interior angles total? 1260? And the sum or the division? 140. Very good. So this is interior angles of polygons. We know how to find this without a table. We can divide up our shape into triangles and then add up the sum of those triangles. So based on something you guys already know.

Allright, thank you very much guys, put your protractors on your card. Calculators, up front. Have a wonderful weekend. Come see me if you want to see your test. If you were absent last time, you need to take the test so. I think they're still absent. Student did you have to take a test?